

Figure 2 | Model simulation of the termination. **a, b**, Optimum simulations are shown as thick lines for the NGRIP scenario (red) and the EPICA Dome C scenario (blue). Uncertainty intervals are shown as the lighter-coloured patches. Also shown are the equilibrium sea levels (thin lines) derived from the ice-core records. The equilibrium sea levels correspond to the ice-core temperature records at a given $\Delta T'$, which is shown on the nonlinear scales to the right of the plots. The sea-level proxy data are marked as black symbols (see Supplementary Fig. S1 for key and references). Sea level is given relative to late Holocene values.

due to ice sheet/climate feedbacks. In defining this function, we need to account for several physical effects that have been shown to be robust features of ice-sheet growth and decay during the last glacial period in both proxy reconstructions and numerical models of ice sheets (see further discussion in Supplementary Information). As a result of these well-documented processes, ice-sheet size and sea level have a tendency to remain close to glacial maximum or minimum, with relatively rapid retreats from maxima, and into and out of minima, triggered by crossing thresholds in the nonlinear system. This hysteresis behaviour has been described in simple models for Northern Hemisphere ice sheets⁸ and for East Antarctica⁹. Quaternary sea-level data over the past 500,000 years support this concept in suggesting two quasi-equilibrium states (interglacial and glacial)^{10–12} that are separated by rapid transitions¹³. Therefore, we represent the equilibrium sea level (S_e) as a function of the inverse hyperbolic sin (\sinh^{-1}) with respect to temperature:

$$S_e = A \sinh^{-1} \left(\frac{\Delta T' + c}{b} \right) + d \quad (1)$$

where $\Delta T'$ is defined as the non-dimensional change in temperature with respect to mean late Holocene temperatures (2 kyr BP

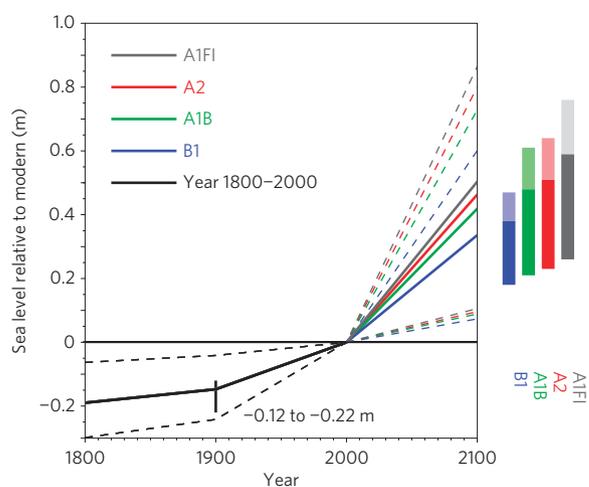


Figure 3 | Model projections of sea-level rise for the twenty-first century.

The best estimate projections are shown as solid lines; the range is shown by the dashed lines. The range of projected sea-level rise takes into account the temperature range from IPCC for individual emissions scenarios, uncertainty in ΔT_{LGM} , which is taken to be 3.3–5.1 °C with a best estimate of 4.2 °C, and uncertainty in the warming estimate from the past century (0.56–0.92 °C, with a best estimate of 0.74 °C; ref. 1). For comparison, IPCC estimates (mean from 2090 to 2100) for individual emissions scenarios are given by the coloured vertical bars on the right. The lighter part represents the extended IPCC (ref. 1) estimate allowing for the extra contribution of 0.09–0.17 m sea-level rise due to accelerated ice-sheet dynamics under a warmer climate. The vertical black line represents sea level in 1900, allowing for a rise in sea level of between 0.12 and 0.22 m over the 1900s. Values are shown in Table 1.

to present). Temperature is non-dimensionalized with respect to ΔT_{LGM} , the difference in temperature between the late Holocene and the LGM, so that $\Delta T' = (T - T_{Holocene}) / \Delta T_{LGM}$. All of the variables used in this letter are summarized and defined in Supplementary Table S1. Note that S_e is constrained to pass through values for the Last Interglacial^{14,15} (LIG; see the Methods section).

Figure 1 shows the best-fit sea-level sensitivity curves that are used in this letter. We note that the \sinh^{-1} function derived here (equation (1)) passes through several independent constraints on the sea-level sensitivity curve. The model results indicate a much reduced sensitivity for warmer temperatures compared with LGM temperatures, implying that the negative feedbacks in operation at the glacial maximum are relatively weak compared with the feedbacks at glacial minimum. Further discussion of the sensitivity of our method for different functions for S_e is given in Supplementary Information.

Next we define a response time, τ (in units of kyr), that defines the rate at which the modelled sea level (S_m) rises in response to an increase in equilibrium sea level in a similar fashion to an earlier study⁶, so that at time t :

$$\frac{dS_m}{dt} = r \cdot \frac{1}{\tau} \cdot [S_e(\Delta T'(t)) - S_m(t)] \times \begin{cases} r = 1, & \text{if } S_e(\Delta T'(t)) > S_m(t) \\ 0 < r < 1, & \text{if } S_e(\Delta T'(t)) < S_m(t) \end{cases} \quad (2)$$

where τ is a typical response time of sea-level rise following a shift in temperature. We have introduced a factor r (varied between 0 and 1) that permits us to take into account the fact that sea-level rise due to ice-sheet decay is faster than sea-level

Table 1 | Model projections of sea-level rise for various IPCC scenarios for the twenty-first century.

	Temperature change (°C)		Published sea-level rise (m)	Model projections of sea-level rise (m)	
	Best estimate*	Range	Range	Best estimate*	Range
Pre industrial				0.04	0.02–0.06
Fleming <i>et al.</i> ²⁸	—	—	0.04–0.06 [†]	—	—
Church <i>et al.</i> ¹⁹	—	—	0.02–0.06	—	—
Domingues <i>et al.</i> ²⁹	—	—	0.02	—	—
1900–2000					
IPCC ¹	0.74	0.56–0.92	0.12–0.22	0.15	0.04–0.24
2090–2100 (mean)					
IPCC ¹ scenario A1FI	4.0	2.4–6.4	0.26–0.59 (0.76 [§])	0.48	0.10–0.82 [‡]
IPCC ¹ scenario A2	3.4	2.0–5.4	0.23–0.51 (0.64 [§])	0.44	0.09–0.76 [‡]
IPCC ¹ scenario A1B	2.8	1.7–4.4	0.21–0.48 (0.61 [§])	0.40	0.08–0.69 [‡]
IPCC ¹ scenario B1	1.8	1.1–2.9	0.18–0.38 (0.47 [§])	0.32	0.07–0.57 [‡]

*Refers to estimates forced by the model optimized values and the mean or 'best estimate' from the IPCC scenarios.

[†]Note that Fleming *et al.*²⁸ suggest a possible reduced rate of sea-level rise over the past 2 kyr compared with the period between 7 and 2 kyr ago.

[‡]The range of projected sea-level rise takes into account the temperature range for individual emissions scenarios¹, uncertainty in ΔT_{LGM} , which is taken to be 3.3–5.1 °C with a mean value of 4.2 °C, and uncertainty in the warming estimate from the past century (0.56–0.92 °C, with a best estimate of 0.74 °C (ref. 1)).

[§]Increased upper limit on range, allowing for increased rise due to uncertainty in the contribution from accelerated ice-sheet dynamics.

fall due to ice-sheet growth¹⁶. In equation (2), the sea-level rise is fastest following a period of large warming, whereas rapid cooling would cause a longer response time τ/r . As we are considering a period dominated by sea-level rise, the model is not sensitive to the precise value of r . We include r for completeness and to consider the possibility of any brief sea-level lowering during the Younger Dryas cold period or the Antarctic cold reversal (see Supplementary Fig. S1, see sensitivity tests in Supplementary Information). Note that the use of a single timescale to represent the integrated contributions of ocean thermal expansion, glacier retreat and changes in ice sheets is an important simplification that limits the useful application of the model to centennial and longer-term responses (see Supplementary Section S6.2).

As r varies between periods of sea-level increase and decrease, it is not possible to solve equation (2) in closed form. Instead sea level is given by integrating (2) over time steps $\Delta t = 0.1$ kyr so that:

$$S_m(t + \Delta t) = S_m(t) + r \cdot \frac{\Delta t}{\tau} \cdot [S_e(\Delta T'(t)) - S_m(t)] \quad (3)$$

where r takes the appropriate value depending on the sign of $S_m(t) - S_e(\Delta T'(t))$. The precise value of time step Δt that is used has no impact on our results.

Equations (1) and (3) require that we define the history of the temperature variability that forced changes in sea level over the last deglaciation. During the last deglaciation, millennial variability affected temperatures differently in the Northern and Southern hemispheres^{17,18}. We therefore consider two temperature proxy alternatives to represent the temperature forcing of sea level during the deglaciation: the oxygen isotope ($\delta^{18}\text{O}$) record of the North Greenland Ice Core Project¹⁷ (NGRIP) (representative of the Northern Hemisphere) and the deuterium (δD) record of the European Project for Ice Coring in Antarctica (EPICA) Dome C (ref. 18) (representative of the Southern Hemisphere) (see Supplementary Fig. S1). We provide an evaluation of each of these scenarios in Supplementary Information. We will call the scenarios: 'the NGRIP scenario' and 'the EPICA Dome C scenario'.

We require some means to evaluate and tune the model output over the past 22 kyr. We define the proportion of the variance

explained by the model (R^2 , see Supplementary Equation S1) by comparing the model estimate with the estimate from sea-level proxies (S_{proxy}). Observational constraints (S_{proxy}) of sea-level rise are based on indicators of past sea level recorded at sites distant from the principal ice sheets ('far-field' sites). Sea-level indicators such as fossil corals or other depth-dependent coastal deposits reflect isostatic effects associated with the ice–water surface mass redistribution as well as variations in global (eustatic) sea level. A full discussion of these data and a note on the effect of isostatic rebound are given in Supplementary Information.

Figure 2 shows the model simulations of the past 22 kyr for both the NGRIP and EPICA Dome C forcing scenarios. These simulations allow for uncertainty in the data and input variables (see the Methods section). Significantly, the NGRIP scenario captures a number of details in the structure of the sea-level record during the different phases of the deglaciation (post-LGM, Bølling–Allerød warming, Younger Dryas cooling, post-Younger Dryas warming and so on), whereas the EPICA Dome C scenario does not (in particular note the deviations during the post-LGM period, the Antarctic cold reversal and the transition to the Holocene). Despite these discrepancies, the R^2 for the optimal solution is only slightly higher for the NGRIP scenario than for the EPICA Dome C scenario (0.99 compared with 0.97). Therefore, we distinguish the success of the NGRIP compared with the EPICA Dome C simulations on the basis of its ability to capture the structure of the sea-level rise. In addition, the best-fit equilibrium sea-level curve (Fig. 1) for the NGRIP scenario passes through the independent estimates from Marine Isotope Stage 3 (MIS 3) and the initiation of the Antarctic ice sheet, whereas the curve for the EPICA Dome C scenario does not. On the basis of these findings, we reject the EPICA Dome C scenario and focus on the NGRIP scenario for the remainder of the letter.

We note that the deglaciation comprises several periods with large changes in sea level on centennial timescales (in particular at the time of the Bølling–Allerød warming and the warming following the Younger Dryas). As these periods of rapid change represent a substantial proportion of the total sea-level rise during the deglaciation, we are confident that the model is capable of resolving sea-level change on centennial timescales. We test this assertion in Supplementary Information where we force

the model with the annual mean temperature record for the twentieth century and compare it with the annual mean sea-level record (see Supplementary Fig. S5). The offsets between the model and data over this period are an order of magnitude smaller than the sea-level rise over the twentieth century and are linked to decadal fluctuations in sea level rather than the centennial trend. We conclude that the model can resolve sea-level rise on centennial timescales (but not decadal), and it is reasonable to approximate the contributions to sea-level rise from multiple sources (that is, glaciers, ice sheets and thermal expansion) with one, integrated model. As the anthropogenic temperature perturbation began around 1900 and possibly earlier⁷, by 2100 this perturbation will have lasted at least two centuries. It is important for this application that the model is tuned to multi-centennial records.

The NGRIP scenario shows continuing sea-level rise of 0.02–0.06 m per century before the industrial period (Figs 2a and 3, Table 1), in close agreement with other estimates derived using independent methods (see Supplementary Table S1). Given that the model successfully simulates the sea-level variability over the past 22 kyr (Fig. 2a) and the twentieth century (see Supplementary Fig. S5), we next use the model to make projections of the model response to future warming.

Over the twenty-first century, projected sea-level rise reaches a maximum of 0.82 m in response to warming from the upper estimate of the A1FI emissions scenario (6.4 °C) and a minimum of 0.07 m rise for the lower warming estimate for the B1 emissions scenario (1.1 °C, Fig. 3 and Table 1). These extremes in our model projections compare to IPCC estimates of 0.59 m for the upper limit of the A1FI scenario and 0.18 m for the lower limit of the B1 scenario¹. Our estimates converge more closely with IPCC estimates when including the extra 0.09–0.17 m rise that the IPCC's fourth assessment report estimated was the potential contribution over the next century from accelerated ice-sheet dynamics¹. A substantial portion of the uncertainty intervals calculated here and those of the IPCC simulations intersect and our results generally increase the confidence that one may have in the IPCC results. As the time constant of the sea-level response is 2,900 years, our model indicates that the impact of twentieth-century warming on sea level will continue many centuries into the future and therefore constitutes an important component of irreversible climate change in the future. Our results indicate the importance of cumulative anthropogenic warming over several centuries on sea-level rise as recognized in IPCC reports¹⁹.

Methods

We define $\Delta T'$ with reference to the NGRIP (ref. 17) and EPICA Dome C (ref. 18) ice-core proxy temperature (T) reconstructions. The value of ΔT_{LGM} for each of the ice-core proxy records is taken as the mean value between 25 and 20 kyr BP so that $\Delta T' = (T - T_{\text{Holocene}}) / \Delta T_{\text{LGM}}$.

In equation (1), the variable b controls the slope of S_e with respect to $\Delta T'$ and c controls the midpoint of the transition in S_e with respect to $\Delta T'$ (see sensitivity study in Supplementary Information). We vary c and b and then scale A so that S_e gives the correct magnitude of change at the LGM (Fig. 1). For the purpose of this study, we assume an LGM sea level of between –120 and –140 m (refs 20, 21). We then adjust d so that S_e passes through the Holocene value (0 m). The variable b controls the difference in S_e between the late Holocene period and the LIG. We randomly vary b so that S_e passes through the sea-level estimates of 3–6 m above modern for the LIG (ref. 14, 15) (Fig. 1). The variable c is controlled by the period of rapid sea-level change associated with the termination. The value of c is left unconstrained and is given by the model optimization.

To constrain b , we need to define $\Delta T'$ for the LIG. We derive the temperature difference between the Holocene period and the LIG (~125,000 years ago), ΔT_{LIG} ($\Delta T_{\text{LIG}} = T_{\text{LIG}} - T_{\text{Holocene}}$), from ice-core temperature constraints (3–5 °C) (ref. 22), assuming a polar amplification factor of two²³ to give the global mean estimate of 1.5–2.5 °C. To obtain $\Delta T'_{\text{LIG}}$, we need to define the global mean ΔT_{LGM} . On the basis of a compilation of model estimates, we take ΔT_{LGM} to be between 3.3 and 5.1 °C, with a preferred estimate of 4.2 °C (ref. 1). Note that by defining $\Delta T'$ and a given polar amplification factor, the warmth of the LIG compared with the Holocene is relative so that the same value represents relative polar warming or the global mean

warming. An obvious concern with using LIG values is that insolation forcing was not the same during the LIG compared with the Holocene. Rohling *et al.*¹² recently showed a close relationship between sea level and temperature over the past 525 kyr on multi-millennial (that is, quasi-equilibrium) timescales, indicating a close coupling of temperature and sea level regardless of large changes in insolation. This justifies our assumption that increased temperature can be considered the primary driver of higher sea level during the LIG.

Using the proxy sea-level data, equations (1) and (3) are optimized to give the maximum value for R^2 using the lsqcurvefit function in the Matlab Optimisation toolbox. The model is initiated at a sea level of –130 m. The model is not sensitive to the sea level at which it is initiated. To avoid sampling bias associated with the increased data density during the Holocene we carry out 10,000 simulations, each time fitting the model to a different, randomly sampled and evenly distributed subset of half of the proxy observations. To account for the uncertainty in the proxy observations, we randomly vary the data in the stated uncertainty range for each sample, assuming that this uncertainty is normally distributed. In this way we are able to estimate each of the variables by taking the mean of each variable weighted to the R^2 value for all of the simulations. We are also able to estimate the uncertainty associated with each of the variables.

For the projections, we force the model with a warming of 0.74 °C over the past century¹ and further warming over the next century from the IPCC emissions scenarios¹ (Fig. 3, Table 1). We allow for the range of uncertainty in warming over the past century of between 0.56 and 0.92 °C and for the uncertainty in the projected warming over the coming century for individual emissions scenarios¹ by carrying out multiple simulations representing the maximum, minimum and preferred warming estimates over the past century and the coming century. In each case the uncertainty in the model variables is included to generate a maximum and minimum estimate of sea-level rise for the next century. We remind the reader that by defining $\Delta T'$ and a given polar amplification factor, any future warming is relative, so that the same $\Delta T'$ is representative of polar warming or the global mean warming.

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